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The Structure of the QCD Potential in $2+1$ Dimensions

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Abstract: We calculate the screening and anti-screening contributions to the inter-quark potential in $2+1$ dimensions, which is relevant to the high temperature limit of QCD. We demonstrate that the relative strength of screening to anti-screening agrees with the $3+1$ dimensional theory to better than one percent accuracy.

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Sparked in part by the discovery of the top quark, there has recently been a great deal of interest in the inter-quark potential, see, for example, [1–7]. It has long been known [8–12] that the pure QCD corrections to the Coulombic potential in 3+1 dimensions are of two types: a dominant anti-screening contribution and a lesser interaction which corresponds to screening by physical, transverse gluons. In this letter we will continue our programme [3, 13] to study the structure of the forces between quarks. We will demonstrate that in $2+1$ dimensions, which for Euclidean metrics is by dimensional reduction related [14,15] to the high temperature limit of QCD, the inter-quark potential has an unexpectedly rich structure and that the relative weights of the attractive and repulsive interactions are almost identical to those of the $3+1$ case.

In $SU(N)$, for static quarks without additional light fermions, the inter-quark potential to order g^4 in $d+1$ space-time dimensions is given by [7]

$$V(q) = -\frac{g^2 C_F}{\boldsymbol{q}^2}\left\{1 + g^2\mu^{2\epsilon}C_A(4d-1)\frac{|\boldsymbol{q}|^{d-3}}{(16\pi)^{\frac{d}{2}}}\frac{\Gamma(\frac{3-d}{2})\Gamma(\frac{d+1}{2})}{\Gamma(\frac{d}{2}+1)}\right\}, \qquad (1)$$

where $q = |\boldsymbol{q}|$, $C_A = N$, $C_F = (N^2-1)/(2N)$ and $d+1 = 4-2\epsilon$. In $3+1$ dimensions this reduces to the familiar form [16]

$$V(q) = -\frac{g^2 C_F}{\boldsymbol{q}^2}\left\{1 - \frac{g^2}{(4\pi)^2}C_A\,\frac{11}{3}\ln\left(\frac{\boldsymbol{q}^2}{\mu^2}\right)\right\}, \qquad (2)$$

while in $2+1$ dimensions this becomes [17]

$$V(q) = -\frac{g^2 C_F}{\boldsymbol{q}^2}\left\{1 + g^2 C_A\frac{7}{32|\boldsymbol{q}|}\right\}. \qquad (3)$$

Note that in $2+1$ dimensions the result is finite and no renormalisation is needed [7]. In $3+1$ dimensions the order g^4 correction to the Coulombic potential is related to the universal beta function of QCD, but in $2+1$ dimensions no such identification is possible since the beta function vanishes. Finally, we recall that in $2+1$ dimensions the coupling constant is a dimensionful quantity.

These corrections to the potential have been understood in 3+1 dimensions as the sum of two distinct physical effects: a dominant anti-screening interaction which arises from the Coulombic potential, and a smaller screening interaction which arises from the virtual production of physical, i.e., gauge invariant, gluon pairs. The dominance of anti-screening over screening is the origin of QCD's asymptotic freedom. Concretely the coefficient of the logarithmic correction can be decomposed as:

$$V(q) = -\frac{g^2 C_F}{\boldsymbol{q}^2}\left\{1 - \frac{g^2}{(4\pi)^2}C_A\,\left[4 - \frac{1}{3}\right]\ln\left(\frac{\boldsymbol{q}^2}{\mu^2}\right)\right\}, \qquad (4)$$

where the factor of 4 comes from the anti-screening interaction and the $\frac{1}{3}$ from the smaller screening forces. The relative strength of the screening part of the potential is, we note, only 8.33% of the anti-screening contribution. Due to the universality of the beta function, this decomposition can be calculated in many different ways [8–12, 18], although it cannot be obtained from the Wilson loop approach to the potential. This structure of the inter-quark potential was previously unknown in $2+1$ dimensions and we shall now calculate it.

We will follow the method of Ref. [3]. The lowest energy states corresponding to two heavy quarks a distance $r = |\boldsymbol{r}|$ apart is $|\bar{\psi}(\boldsymbol{r})h(\boldsymbol{r})h^{-1}(0)\psi(0)\rangle$, where the quarks are in the same time slice. We call h^{-1} a dressing for the matter field, ψ. This field dependent term is the lowest energy gluonic configuration around an individual fermion which maintains gauge invariance for the composite charged quark. The kinematics of the heavy quark determines the form of the dressing, and we have shown elsewhere [19] that it factors into a product of two terms: a gauge dependent term which makes the dressed quark gauge invariant, and a gauge invariant structure.

The first part of the dressing is the minimal gluonic configuration which renders the quark gauge invariant. This first term originates from Gauss' law and hence from longitudinal degrees of freedom. It produces the spreading of the colour charge, *anti-screening*, which underlies asymptotic freedom in non-abelian gauge theories, and will raise the energy of the quark-antiquark state. It is the non-abelian extension of the Coulomb interaction. Since the overall dressed quark has to correspond to the lowest energy state, the additional gauge invariant glue must lower the energy. As such, it can only correspond to a *screening* contribution. This physical decomposition into structures which necessarily raise and lower the energy is the correct identification of anti-screening and screening effects even in 2+1 dimensions where the coupling does not run.

Generalising the construction in [3] to d-spatial dimensions, the anti-screening part of the potential at order g^4 is

$$\begin{aligned} V^{(4)}_{\text{anti}}(r) &= -3g^4 C_A C_F \frac{\Gamma^3(\frac{d}{2}-1)}{64\pi^{\frac{3d}{2}}} \int \mathrm{d}^d z\, \mathrm{d}^d w \frac{1}{|\boldsymbol{z}-\boldsymbol{w}|^{d-2}} \times \\ & \qquad \left(\partial_j^z \frac{1}{|\boldsymbol{z}-\boldsymbol{r}|^{d-2}}\right)\left(\partial_k^w \frac{1}{|\boldsymbol{w}|^{d-2}}\right)\langle 0|A_j^{\mathrm{T}}(\boldsymbol{z})A_k^{\mathrm{T}}(\boldsymbol{w})|0\rangle\,, \end{aligned} \tag{5}$$

where we have used the d-dimensional result

$$\left(\frac{1}{\nabla^2}f\right)(x^0,\boldsymbol{x}) = -\frac{\Gamma(\frac{d}{2}-1)}{4\pi^{\frac{d}{2}}}\int \mathrm{d}^d z \frac{f(x^0,\boldsymbol{z})}{|\boldsymbol{z}-\boldsymbol{x}|^{d-2}}\,. \tag{6}$$

This reduces to Eq. 16 of [3] for $d=3$. The gauge invariant, equal-time, free propagator in (5), in $d+1$ dimensions, is given by

$$\langle 0|A_j^{\mathrm{T}}(\boldsymbol{z})A_k^{\mathrm{T}}(\boldsymbol{w})|0\rangle = \frac{\Gamma(\frac{d+1}{2})}{2\pi^{\frac{d+1}{2}}}\frac{(z-w)_j(z-w)_k}{|\boldsymbol{z}-\boldsymbol{w}|^{d+1}}\,, \tag{7}$$

which can be understood as the Coulomb propagator for the spatial components or, in the Lorentz class of gauges, as the $\xi = -1$ propagator (where $\xi = 1$ corresponds to Feynman gauge). This last identification, based upon the requirement of the propagator being transverse to ∂^x_i, allows us to use the computational power of covariant gauges. The dimension-independence of this gauge should be contrasted with the Yennie gauge, where the propagator is transverse to momentum derivatives *only* in $d = 3$.

Combining these expressions results in a standard, finite integral which may be straightforwardly evaluated in d dimensions. In $2+1$ dimensions, after Fourier transforming, we have

$$V^{(4)}_{\rm anti}(q) = -\frac{3}{2} g^4 C_A C_F \frac{1}{\boldsymbol{q}^2} \int \frac{\mathrm{d}^2 l}{(2\pi)^2} \frac{1}{|\boldsymbol{l}|(\boldsymbol{l}-\boldsymbol{q})^2} \left(1 - \frac{(\boldsymbol{q}\cdot\boldsymbol{l})^2}{\boldsymbol{q}^2\,\boldsymbol{l}^2}\right) . \tag{8}$$

This is, of course, a *finite* integral. From this we can rapidly show that the anti-screening contribution to the potential at order g^4 is given by

$$V^{(4)}_{\rm anti}(q) = -g^4 C_F C_A \frac{3}{4\pi} \frac{1}{|\boldsymbol{q}|^3} . \tag{9}$$

Comparing this with the full potential in $2+1$ dimensions at this order (3) we see first of all that the factors of π do not agree. However, the energy is still higher than the total result of (3) so it is still a physically acceptable result. We now want to show how the screening contribution supplies the different π factors needed to lower the energy to the final physical result. We will, therefore, now independently calculate the screening contribution.

We shall follow an approach to the potential which has been presented by Gribov [10] and by Drell [11]. Working now in Coulomb gauge, the Hamiltonian is

$$H = \frac{1}{2} \int \mathrm{d}^2 x \left((\boldsymbol{E}^a_{\rm T})^2 + (\boldsymbol{B}^a)^2 - \phi^a \nabla^2 \phi^a \right) , \tag{10}$$

where we have decomposed the chromoelectric field into transverse and longitudinal components, $\boldsymbol{E}^a = \boldsymbol{E}^a_T - \boldsymbol{\nabla}\phi^a$ and summation over colour is understood. Gauss' law tells us that ϕ is related to the static matter sources, ρ, and the gluonic fields by

$$\nabla^2 \phi^a = g \left(\rho^a - f_{abc} \boldsymbol{A}^b \cdot \boldsymbol{E}^c \right) , \tag{11}$$

from which we can obtain the following equation up to order g^3, which is all we shall require in this letter:

$$\nabla^2 \phi^a = \left\{ g\delta^{ae} + g^2 f_{abe} \boldsymbol{A}^b \cdot \boldsymbol{\nabla} \frac{1}{\nabla^2} + g^3 f_{abc} f_{cde} \boldsymbol{A}^b \cdot \boldsymbol{\nabla} \frac{1}{\nabla^2} \boldsymbol{A}^d \cdot \boldsymbol{\nabla} \frac{1}{\nabla^2} \right\} \left(\rho^e - f_{egh} \boldsymbol{A}^g \cdot \boldsymbol{E}^h_{\rm T} \right) , \tag{12}$$

where ∇^2 acts on whatever is on its right. We take the sources for simplicity to have the form $\rho^a = \rho^a_q + \rho^a_{\bar{q}}$ where $\rho^a_q(\boldsymbol{x}) = t^a_q \delta^3(\boldsymbol{x})$, $\rho^a_{\bar{q}}(\boldsymbol{x}) = t^a_{\bar{q}} \delta^3(\boldsymbol{x}-\boldsymbol{r})$. Here we assume that t^a_q and $t^a_{\bar{q}}$ are the colour charges of a heavy (classical) quark q_i and antiquark $\bar{q}_j$ in a normalized colour singlet state $|\Psi\rangle = N^{-1/2}|q_i\rangle|\bar{q}_i\rangle$. Hence the colour factor becomes

$$t^a_q t^a_{\bar{q}} = -\frac{1}{N}\langle q_i|Q^a|q_j\rangle\langle \bar{q}_i|Q^a|\bar{q}_j\rangle = \frac{1}{N}\mathrm{tr}\,(T^a T^a) = -C_F, \tag{13}$$

where Q^a is the colour charge operator and the anti-Hermitian generators T^a are in the fundamental representation of $SU(N)$. The heavy quark and antiquark are again separated by $\boldsymbol{r}$. The sources only enter the Hamiltonian (10) in the last term, so the $\boldsymbol{r}$ dependent term here gives the potential between them.

Let us first explain how the lowest order result may be recovered in this approach. The relevant term in the Hamiltonian is

$$-\frac{1}{2}\int \mathrm{d}^2x\, \phi^a \nabla^2 \phi^a = -g^2 \int \mathrm{d}^2x\, \rho_{\bar{q}}(x)\frac{1}{\nabla^2}\rho_q(x)\,, \tag{14}$$

where, as we are only interested in the potential, we have dropped separation independent terms. We may now evaluate the expectation value between the gluonic vacuum states. Expressing the delta functions as Fourier transforms and trivially performing the spatial integral, we obtain

$$V(q) = -g^2 C_F \frac{1}{\boldsymbol{q}^2}, \tag{15}$$

from which we can read off the three dimensional generalisation of the Coulomb interaction, i.e., the lowest order term in (3).

We may now proceed to the g^4 contributions. From time independent perturbation theory we may write it as the sum of anti-screening and screening effects, $V^{(4)}(r) = V^{(4)}_{\mathrm{anti}}(r) + V^{(4)}_{\mathrm{scr}}(r)$, where

$$V^{(4)}_{\mathrm{anti}}(r) = -\frac{1}{2}\int \mathrm{d}^2x\, \langle 0|\phi^a \nabla^2 \phi^a|0\rangle, \tag{16}$$

$$V^{(4)}_{\mathrm{scr}}(r) = \frac{-1}{4}\sum_{n\neq 0}\frac{1}{E_n}\int \mathrm{d}^2x\, \langle 0|\phi^a \nabla^2 \phi^a|n\rangle \int \mathrm{d}^2x\, \langle n|\phi^b \nabla^2 \phi^b|0\rangle\,, \tag{17}$$

and E_n is the energy of the state $|n\rangle$. In the second term it is sufficient, at this order, to sum over a complete set of intermediate states of two transverse gluons.

These two terms again represent the two distinct physical interactions that occur in QCD: the first is the non-abelian generalisation of the Coulombic interaction, while the second describes the exchange of physical, transverse gluons. This second term, since it

comes from the exchange of physical quanta, represents the expected screening part of the potential which lowers the interaction energy.

Let us first verify that this anti-screening contribution agrees with our previous result. At this order we need to retain terms up to order g^3 in ϕ. From (12) we obtain three identical contributions

$$V^{(4)}_{\rm anti}(r) = -\frac{3}{2} g^4 f_{abc} f_{cde} \int \mathrm{d}^2x \, \langle 0|\rho^a \frac{1}{\nabla^2} \boldsymbol{A}^b \cdot \boldsymbol{\nabla} \frac{1}{\nabla^2} \boldsymbol{A}^d \cdot \boldsymbol{\nabla} \frac{1}{\nabla^2} \rho^e |0\rangle \,. \tag{18}$$

This result is precisely Eq. 5 as expected.

To find the screening contribution, we now need to insert physical two gluon states into (17). The sum over such states then becomes a sum over colour (e, f) and helicity (λ, σ) and an integral over momenta. Explicitly we have:

$$\sum_{n=2 \text{ gluon}} |n\rangle\langle n| = \frac{1}{2} \sum_{e\,f} \sum_{\lambda\,\sigma} \int \mathrm{d}^2k \int \mathrm{d}^2l \; a^\dagger_e(\lambda, \boldsymbol{k}) a^\dagger_f(\sigma, \boldsymbol{l}) |0\rangle\langle 0| a_f(\sigma, \boldsymbol{l}) a_e(\lambda, \boldsymbol{k}) \,. \tag{19}$$

The only terms from (12) that can contribute to these transverse states are given by $\phi^a \nabla^2 \phi^a \to -2g^2 f_{abc}\, \rho^a \nabla^{-2} \boldsymbol{A}^b \cdot \boldsymbol{E}^c_{\rm T}$. We may thus write

$$V^{(4)}_{\rm scr}(r) = -2g^4 \sum_{n=2 \text{ gluon}} \frac{1}{E_n} \int \mathrm{d}^2x f_{abc} \, \langle 0|\rho^a_q \frac{1}{\nabla^2} \boldsymbol{A}^b \cdot \boldsymbol{E}^c_{\rm T} |n\rangle \int \mathrm{d}^2w f_{def} \, \langle n|\rho^d_{\bar q} \frac{1}{\nabla^2} \boldsymbol{A}^e \cdot \boldsymbol{E}^f_{\rm T} |0\rangle \,, \tag{20}$$

and, in Coulomb gauge, we may identify the transverse electric field with $-\dot{A}_i$. Using the standard commutator, $[a_b(\lambda, \boldsymbol{k}), a^\dagger_c(\sigma, \boldsymbol{l})] = \delta_{bc}\delta_{\lambda\sigma}\delta^3(\boldsymbol{k} - \boldsymbol{l})$ we then rapidly obtain

$$\begin{aligned} V^{(4)}_{\rm scr}(q) &= \frac{g^4}{4} C_A C_F \frac{1}{|\boldsymbol{q}|^4} \int \frac{\mathrm{d}^2l}{(2\pi)^2} \int \mathrm{d}^2k \frac{\delta^3(\boldsymbol{q} - \boldsymbol{k} - \boldsymbol{l})(|\boldsymbol{l}| - |\boldsymbol{k}|)^2}{|\boldsymbol{l}|\,|\boldsymbol{k}|\,(|\boldsymbol{l}| + |\boldsymbol{k}|)} \\ &\quad \times \sum_\lambda \epsilon^i(\lambda, \boldsymbol{k}) \epsilon^j(\lambda, \boldsymbol{k}) \sum_\sigma \epsilon^i(\sigma, \boldsymbol{l}) \epsilon^j(\sigma, \boldsymbol{l}) \,, \end{aligned} \tag{21}$$

where we have already carried out the $\boldsymbol{x}$ integral and some trivial momentum integrals. We now exploit the Coulomb gauge relation

$$\sum_\lambda \epsilon^i(\lambda, \boldsymbol{k}) \epsilon^j(\lambda, \boldsymbol{k}) = \delta^{ij} - \frac{k^i k^j}{\boldsymbol{k}^2} \,, \tag{22}$$

to arrive at the final expression for the screening contribution

$$V^{(4)}_{\rm scr}(q) = \frac{g^4}{4} C_A C_F \frac{1}{|\boldsymbol{q}|^4} J(q) \,, \tag{23}$$

where

$$J(q) = \int \frac{\mathrm{d}^2l}{(2\pi)^2} \frac{(|\boldsymbol{l}| - |\boldsymbol{q} - \boldsymbol{l}|)^2}{|\boldsymbol{l}|\,|\boldsymbol{q} - \boldsymbol{l}|\,(|\boldsymbol{l}| + |\boldsymbol{q} - \boldsymbol{l}|)} \left\{ 1 - \frac{\boldsymbol{q}^2}{(\boldsymbol{q} - \boldsymbol{l})^2} + \frac{(\boldsymbol{q} \cdot \boldsymbol{l})^2}{\boldsymbol{l}^2\,(\boldsymbol{q} - \boldsymbol{l})^2} \right\} \,, \tag{24}$$

The term $|\boldsymbol{l}|+|\boldsymbol{q}-\boldsymbol{l}|$ in the denominator is unusual and makes this a more difficult integral[4].

To evaluate this integral it is convenient to go into polar co-ordinates, ρ,θ, and then make the change of variables: $\rho=(\tau^2-1)/[2(\tau-\cos\theta)]$. The angular integral may then be performed and afterwards it is not too difficult to evaluate the integral over τ. The result is

$$J(q)=\left(-\frac{7}{8}+\frac{3}{\pi}\right)|\boldsymbol{q}|\,. \tag{25}$$

We so obtain for the screening contribution to the interquark potential in 2+1 dimensions

$$V^{(4)}_{\rm scr}(q)=-g^4C_FC_A\frac{1}{4|\boldsymbol{q}|^3}\left(\frac{7}{8}-\frac{3}{\pi}\right)\,. \tag{26}$$

This, together with the anti-screening result (9), gives us the total order g^4 contribution to the interquark potential in $2+1$ dimensions:

$$V^{(4)}(q)=-g^4C_FC_A\frac{1}{4|\boldsymbol{q}|^3}\left[\frac{3}{\pi}-\left(\frac{3}{\pi}-\frac{7}{8}\right)\right]\,. \tag{27}$$

We see that, as expected, the sum of the dominant anti-screening contribution and this screening term gives exactly the correct result for the total potential (3). As had to be the case, the various factors of π have combined to give one overall factor. This physical decomposition cannot be seen in the Wilson loop approach. There only the contributions from different classes of diagrams can be distinguished, however, they are gauge dependent (and all have the same π factors).

The relative numerical weighting of the screening and anti-screening contributions to the potential is now 8.37%. This is remarkably within one part in a hundred of the split in $3+1$ dimensions!

There is a pressing need for a detailed understanding of the structure of the forces in QCD. In this paper we have calculated the screening and anti-screening contributions to the static inter-quark potential in $2+1$ dimensions:

$$V(r)=\frac{g^2C_F}{2\pi}\ln(g^2r)+\frac{g^4C_FC_A}{8\pi}\left[\frac{3}{\pi}-\left(\frac{3}{\pi}-\frac{7}{8}\right)\right]r\,. \tag{28}$$

This calculation is of interest in itself, given that the beta function now vanishes, in that it supports the idea that the $2+1$ theory models many of the important features of full QCD. We have seen that, to a first approximation, it is safe to neglect gluonic

[4]We need to calculate the full, finite integral, while in $3+1$ dimensions, where we have the same integrand, we only need to extract the logarithmic divergence, so this denominator term effectively reduces to $2|\boldsymbol{l}|$ and the calculation is trivial.

screening effects in $2+1$ dimensions. Additionally we note that it is important to understand the $2+1$ dimensional theory, since it is related by dimensional reduction to the high temperature limit of QCD (although this identification is not direct [20]). The physical decomposition that we have calculated exhibits a curious mathematical property (the differing transcendental factors) and the unexpected physical behaviour that the relative weights of screening and anti-screening are nearly identical in both $2+1$ and $3+1$ dimensions.

We note that the one loop anti-screening coefficient of the linearly rising term is only 1.35 times the lattice result in 2+1 dimension [21]. It is not clear to us why there is such good agreement, nor how a linear potential emerges from the lattice when higher perturbative corrections will have the form of a power series in r.

We now want to extend this work in various directions. The separation into screening and anti-screening is not known at higher orders in the coupling. As noted by Drell [11], the method used in the latter part of this paper does not easily lend itself to such calculations. Our approach, based on a manifestly gauge invariant construction of quarks and gluons, can be readily extended to higher orders (see the appendix of [13]). Indeed we have previously shown [19] in QED that such dressed fields have infra-red finite on-shell Green's functions at all orders in perturbation theory. (The same decomposition of the dressing into a minimal and a separately gauge invariant part is also reflected in the infra-red structures of QED.) We are thus in the process of calculating the, hitherto unknown, decomposition of the potential into screening and anti-screening effects at order g^6 in both $3+1$ and $2+1$ dimensions. Another important extension of this work is to repeat the $3+1$ calculation at finite temperature. The results of this letter could be taken as indicating that the anti-screening/screening decomposition is insensitive to the temperature. If this is indeed the case, one needs to discover what aspect of strong interaction physics underlies this remarkable property.

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